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# DD–763

## M. A./M. Sc. (Fourth Semester) EXAMINATION, 2020

MATHEMATICS

Paper Second

**(Partial Differential Equations and Mechanics)**

*Time : Three Hours*

*Maximum Marks : 80*

**Note :** All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

### Unit—I

1. (a) State and prove local existence theorem for first order non-linear PDE.

   (b) Prove that the function $x$ (.) solves the system of Euler-Lagrange's equations :

$$-\frac{d}{ds}\;\mathrm{D}_q\mathrm{L}\,(\dot{x}(s), x(s)) \;+ \mathrm{D}_x\mathrm{L}\;\; \dot{x}(s), x(s) \;\; = 0$$

$$(0 \le s \le t)$$

   (c) State and prove Lax-Oleinik's formula.

**P. T. O.**

**Unit—II**

2. (a) Using the method of separation of variables solve the heat equation :

$$u_t - \Delta u = 0 \text{ in } \mathrm{U} \times (0, \infty)$$

$$u = 0 \text{ on } \partial \mathrm{U} \times [0, \infty)$$

$$u = g \text{ on } \mathrm{U} \times \{t = 0\}$$

where $g : \mathrm{U} \to \mathrm{R}$ is given.

(b) Derive Barenblott's solution to the porous medium equation using similarity under scaling.

(c) Explain Hodograph transform.

**Unit—III**

3. (a) Suppose $k, l : \mathrm{R} \to \mathrm{R}$ are continuous functions, that $l$ grows at most linearly and that $k$ grows at least quadratically. Assume also there exists a unique point $y_0 \in \mathrm{R}$ such that :

$$k(y_0) = \min_{y \in \mathrm{R}} k(y)$$

Then prove that :

$$\lim_{\epsilon \to 0} \frac{\int_{-\infty}^{\infty} l(y)\, e^{-\frac{k(y)}{\epsilon}}\, dy}{\int_{-\infty}^{\infty} e^{-\frac{k(y)}{\epsilon}}\, dy} = l(y_0).$$

(b) Discuss unperturbed PDE :

$$\operatorname{div}(u\, b) = \delta_0 \text{ in } \mathrm{R}^2$$

where $\delta_0$ is the Dirac measure $\mathrm{R}^2$ giving unit mass to the point O.

(c) Solve the wave equation using stationary phase method.

## Unit—IV

4. (a) Derive mathematical expressions for Hamilton's principle.

(b) Derive Whittaker's equations.

(c) State and prove Lee Hwa-Chung's theorem.

## Unit—V

5. (a) Derive Hamilton-Jacobi's equations.

(b) Prove that the Lagrange bracket is invariant under canonical transformation.

(c) Using Poisson brackets relation, show that the following transformation is canonical if $ad - bc = 1$ :

$$Q = aq + bp$$
$$P = cq + dp$$