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Total No. of Sections : 03
Total No. of Printed Pages : 03

Code No. : 02/303(B)

Second Semester Examination, May-2018

M.Sc. MATHEMATICS

Paper - III

GENERAL AND ALGEBRAIC TOPOLOGY

Time : 3 Hrs. **Max.Marks : 80**

Note : Section 'A' consists of 10 very short answer type questions, all of which are compulsory and should be attempted first. Section 'B' consists of four short answer type questions with internal options. Section 'C' consists of four long answer type questions with internal choice.

Section - 'A'

Answer the following very short-answer-type questions in one or two sentences : (2×10=20)

Q.1 Define Tychonoff product topology in terms of standard subbase.

Q.2 State Tychonoff's theorem.

Q.3 Define Tychonoff embedding.

Q.4 Define locally finite space.

Q.5 State the Urysohn metrization theorem.

Q.6 Define net.

Q.7 Define ultra filter.

Q.8 Define convergence of nets.

Q.9 Define homotopy path.

Q.10 Define covering spaces.

P.T.O.

Section - 'B'

Answer the following : **(5 4=20)**

Q.1 Prove that the product of two connected spaces is connected.

OR

Prove that the product of two second countable spaces is second countable.

Q.2 Let be a locally finite collection of subsets of *X*. Then prove that any subcollection of *A* is locally finite.

OR

Prove that every closed subspace of a paracompact space is paracompact.

Q.3 Prove that in a Hausdorff space X, every net in X. converge to at most one point.

OR

Prove that a topological space X is compact if every ultrafilter on X converges.

Q.4 Prove that the relation of homotopy is transitive.

OR

Prove that the fundamental group of S' is isomorphic to the additive group of integers.

Section - 'C'

Answer the following : **(10 4=40)**

Q.1 Prove that the product space is Hausdorff iff each coordinate space X_α is Hausdorff.

OR

Prove that the product space is connected iff each coordinate space X_α is connected.

Q.2 State and prove the Nagata-Smirnov metrization theorem.

OR

State and prove the Smirnov metrization theorem.

Q.3 Let be a topological space and let . Then prove that *Y* is -open iff no net in *X*-*Y* can converge to a point in *Y*.

OR

Let *A* be any non-empty family of subsets of a set *X* then prove that there exists a filter on *X*, containing A iff A has the FIP.

Q.4 Prove that the map given by the equation $p(x)=(\cos 2\pi x, \sin 2\pi x)$ is surjective, local homeomorphism but not a covering map.

OR

For a given non-vanishing vector field on B^2, prove that there exists a point of S' where the vector field points directly inward and a point of S' where it points directly outward.

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