

Roll No. ....

# ED–2808

**M. A./M. Sc. (Final) EXAMINATION, 2021**

MATHEMATICS

**(Optional)**

Paper Third (*i*)

**(Graph Theory)**

*Time : Three Hours*

*Maximum Marks : 100*

**Note :** All questions are compulsory. Solve any *two* parts from each questions. All questions carry equal marks.

## **Unit—I**

1. (a) Prove that if a Graph  $H$  is homeomorphic to a Graph  $G$ , then  $G$  is a contraction of  $H$ .
- (b) Show that any Homeomorphism is the product of a connected and a discrete homomorphism.
- (c) Write short notes on the following :
  - (i) Spectrum properties
  - (ii) Cycle space and bond space

**P. T. O.**

**Unit—II**

2. (a) Show that every planar Graph is  $K$ -vertex colorable if every plane graph is  $K$ -face colorable.
- (b) Prove that any uniquely  $K$ -colorable Graph is  $(K-1)$  connected.
- (c) Write short notes on the following :
  - (i) Achromatic and Adjoint numbers
  - (ii) The Rosenfeld numbers

**Unit—III**

3. (a) Prove that every comparability graph is perfect.
- (b) Define interval Graph Show that every interval graph is Triangulated.
- (c) Write short notes on the following :
  - (i) Ramsey numbers and Ramsey graphs
  - (ii) Forbidden Subgraph orientation.

**Unit—IV**

4. (a) Show that every Group is isomorphic to the automorphism group of some graph.
- (b) Prove that every vertex of a composite connected graph lies on a 4-cycle.
- (c) Write short notes on the following :
  - (i) Graph enumeration
  - (ii) Co-chromatic graph

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**Unit—V**

5. (a) Prove that every digraph without odd cycles has a 1-basis.
- (b) Prove that a strong tournament contains cycles of all length  $6$ ,  $3 \leq C \leq n$
- (c) Write short notes on the following :
  - (i) Types of connectedness
  - (ii) Digraphs and Networks.