ED-2801

M.A./M.Sc. (Previous) Examination, 2021

MATHEMATICS

Paper - I

Advanced Abstract Algebra

Time : Three Hours] [*Maximum Marks* : 100

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Define composition series and equivalence of two composition series. Also show that Jordan Holder theorem fails for infinite group by giving example of set of integers.

(*b*) Define finitely generated extension and show that the field of characteristic zero and finite fields are perfect fields.

(*c*) Define normal extension and show that if E is simple extension of F, then there are only a finite number of intermediate fields between F and E.

Unit-II

2. (*a*) Define Galois extension and fixed field also show that the group $G\left(\frac{\mathbb{Q}(2)}{\mathbb{Q}}\right)$, $\alpha^5 = 1$, $\alpha \neq 1$ is isomorphic to the cyclic group of order 4.

(*b*) Define group of F-automorphisms of E and show that if F is the field of characteristic $\neq 2$, with $x^2 - a \in F[x]$ be an irreducible polynomial over F then order of its Galois group is 2.

(*c*) Define solvable Galois group and show that if $f(x) \in F[x]$ is solvable by radicals over F then its splitting field E over F has solvable Galois group $G(E/F)$.

Unit-III

3. (*a*) Define simple module with example and show that an R-module M is cyclic if and only if M is isomorphic to R/I, where I is a left module of R and R is ring with unity.

(*b*) State and prove Hilbert basis theorem.

(*c*) Define Artinian module with example and show that M is noetherian if and only if every submodule of M is finitely generated.

Unit-IV

4. (*a*) Define nilpotent transformation and show that two linear transformations are equivalent if and only if they have same number of invariants.

(*b*) Find Jordan Canonical form of

$$\begin{bmatrix} 0 & 2 & -1 \\ -3 & 8 & 3 \\ 2 & 4 & -1 \end{bmatrix}$$

(*c*) In a vector space V define a transformation T by

$$T(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3) = \alpha_0 + \alpha_1(x+1) + \alpha_2(x+1)^2 + \alpha_3(x+1)^3.$$

Compute the matrix of T is the basis $1,\ 1+x,\ 1+x^2,\ 1+x^3$.

Unit-V

5. (*a*) Define equivalent matrices and show that any submodule of any free R-module is also free and no. of elements is the basis in the submodule are less than or equal to the number of elements in the basis in the module.

(*b*) Define invariants and show that A is equivalent to the diagonal matrix diag $(a_1, a_2, \ldots a_i, 0, 0, 0 \ldots 0)$ such that $a_1 \mid a_2 \mid a_3 \mid \ldots \mid a_i$, where A is an $m \times n$ matrix over a principal ideal domain R.

(*c*) Define Rational Canonical form of matrix. Explain briefly is $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ a diagonal matrix.