OR

Show that if there exists such that

.

Q.4 D. Let $F \in H\left(U-\{0\}\right), F$ be one to one in U, F has a pole of order 1 at , with residue 1, and neither nor are in then show that . **(12)**

OR

If be an analytic function in a region containing the closure of the and satisfying . Then show that there is a dis in which is one-one and such that contains a disc of radius .

---x---

Roll No......................

Total No. of Units : 04
Total No. of Printed Pages : 04

Code No. : 02/403

Second Semester Examination, May 2019

M.Sc. MATHEMATICS

Paper - IV

ADVANCED COMPLEX ANALYSIS (II)

Time : 3 Hrs. **Max. Marks : 80**

- Part A and B of each question in each unit consist of very short answer type questions which are to be answered in one or two sentences.

  Part C is short answer type and Part D is long answer type.

Unit - I

72

Q.1 A. Define Euler's Gamma function. **(2)**

Q.1 B. Write the statement of Euler's theorem. **(2)**

Q.1 C. Prove that the zeta can be extended to a mesomorphic in the whole plane with only a simple pole at $Z=1$ and for satiestfies Riemnn's functional equation. **(4)**

OR

Let U and V be two open subset of with and . If it is a component of and then show that .

Q.1 D. State and prove Legendre's duplication formula. **(12)**

P.T.O.

OR

If Re then show that .

Unit - II

Q.2 A. Write the statement of meanvalue theorem for Harmonic functions. **(2)**

Q.2 B. Define Poisson Kernel. **(2)**

Q.2 C. Let $f(z)$ be analytic in a domain D and let vanish over a domain D_o which is a part of D.Then show that vanish over the whole domain D. **(4)**

OR

Let be a path from a to b and let

and be analytic continuations along r such

that then show that $[f_1]\, b = [g_1]\, b$.

Q.2 D. State and prove Schwarz's Reflection Principle for symmetric region. **(12)**

OR

Show that when $0 < b < 1$ the series

$+ \frac{z-ib}{1+ib} - \frac{1}{2}\frac{(z-ib)^2}{(z+ib)^2} + ---$ is analytic continuation of the function defined by the series $z - \frac{1}{2}z^2 + \frac{1}{3}z^3 ---$.

Unit - III

Q.3 A. Define Rank of an Entire function. **(2)**

Q.3 B. Define Green's function. **(2)**

Q.3 C. Find the order of cos z function. **(4)**

OR

If is an entire function of order and convergence exponent , then show that show that .

Q.3 D. State and prove Hadamrd's three circles theorem. **(12)**

OR

[illegible] Let be analytic for . Let be the moduli of the zero of $f(z)$ in arranged as a non-decreasing sequence. Then show that if , prove that

$$\log \frac{r^n |f(0)|}{r_1, r_2 --- r_n} = \frac{1}{2\pi}\int_0^{2\pi} \log \left| f\left(re^{i\theta}\right) \right| d\theta .$$

Unit - IV

Q.4 A. Define Landau's constant. **(2)**

Q.4 B. Write the statement of $\perp -$theorem. **(2)**

Q.4 C. Let be an analytic function in the disc such that for all z in then show that f is one-one. **(4)**

P.T.O.