Q-2. Prove that the complement of a measurable set is measurable.

OR

Let A be any set and a finite sequence of disjoint measurable sets, then prove that :

$$m^*\left(A \cap \left[\sum_{\Sigma=1}^{n} E_i\right]\right) = \sum_{\Sigma=1}^{n} m^*(A \cap E_i)$$

Q-3. Let (X, B, μ) be a measure space. If , then prove that :

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu(E_n)$$

OR

Let (X, S, μ) be a finite measure space a semiring of sets such that and a measure on if on S then prove that on Σ.

Q-4. Prove that for is a normed linear space with the norm defined by $\|f\|_p = \left(\int_x |f|^p \, d\mu\right)^{1/p}$.

OR

State and prove Holder's Inequality for L^p – spaces.

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Roll No......................

Total No. of Section : 03

Total No. of Printed Pages : 04

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Second Semester Examination, May-2017

M.Sc. MATHEMATICS

Paper - II

REAL ANALYSIS (II)

Time : 3 Hrs. **Max.Marks : 80**

Note : Section 'A', consists of 10 very short answer type questions, all of which are compulsory and should be attempted first. Section 'B' consists of four short answer type questions with internal options. Section 'C' consists of four long answer type questions with internal choice.

Section-'A'

[illegible] $E_i \subset E_{i+1}$

Answer the following very short-answer-type questions **(2x10=20)**

Q-1. Define Riemann Stieltjes sum.

Q-2. Define integration of vector valued function.

Q-3. State mean value theorem.

Q-4. Define Lebesgue outer measure.

Q-5. Define F_σ-set and -set.

Q-6. Define Lebesgue Integral.

Q-7. Define measurable space.

Q-8. Define convex function.

Q-9. State Holder's Inequality.

Q-10. Define almost uniform covergence.

P.T.O.

Section-'B'

Answer the following short-answer-type questions
(5x4=20)

Q-1. State and prove the fundamental theorem of calculus.

OR

If maps into , if for some monotonically increasing function α on then prove that and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$

Q-2. Prove that if E_1 and are measurable sets, then so is .

OR

Let f and be measurable real valued functions defined on a measurable set x. Let be real and continuous on R^2 and put . Then prove that h is measurable.

Q-3. Prove that the function $f(x) = \frac{d}{dx}\left(x^2 \sin \frac{1}{x^2}\right)$

is not Lebesgue integrable over $[0,1]$.

OR

Let be a bounded function defined on . If is Riemann integrable on , then prove that it is Lebesgue integrable on and .

Q-4. State and prove Minkowski's inequality.

OR

Prove that let μ be a positive measure on a -algebra in a set so that . If is real valued function in . If for all and if is convex on

[illegible]

Section-'C'

Answer the following long-answer-type questions (10x4=40)

Q-1. Let f be bounded function, and be a monotonically increasing function on , then prove that on $[a,b]$ if and only if for every there exists a partition P such that .

OR

Let be a curve. If $c \in (a,b)$ then prove that $\wedge_\gamma(a,b) = \wedge_\gamma(a,c) + \wedge_\gamma(c,b)$.